# Frequency Modulation, Part Deux

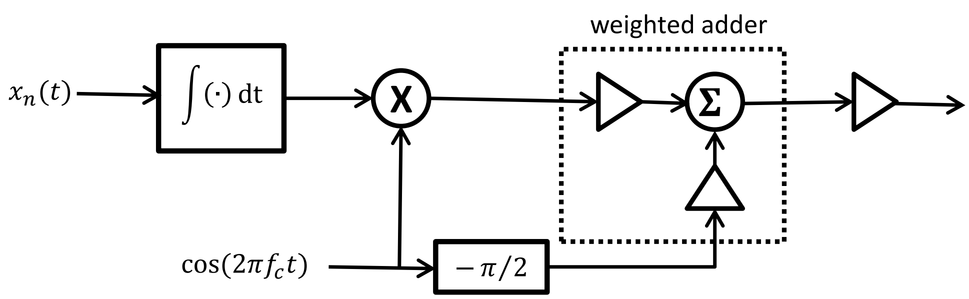
Modules: Audio Oscillator, Wideband True RMS Meter, Twin Pulse Generator, Multiplier, Adder, Phase Shifter, Digital Utilities, 100-kHz Channel Filters, Laplace Transform, FM Utilities, Noise Generator

## 0 Pre-Laboratory Reading

### 0.1 Armstrong Modulator and Narrowband FM

In the previous experiment, you generated FM by the direct method (that is, using a VCO). Here an indirect method of generating FM is explained. In this indirect method, the message signal is integrated; this produces a new signal whose derivative is the original message signal. Phase modulation of this new signal onto the carrier produces FM.

Here a special type of phase modulator, called the *Armstrong modulator*, is used. This phase modulator is only an approximation to a true phase modulator; however, it is a good approximation for small modulation indices. The combination of an integrator and an Armstrong modulator is illustrated below; this represents an approximation to a frequency modulator. The weighting factors in the weighted adder are shown explicitly as amplifiers. In the TIMS weighted adder, the gains of both of these amplifiers are negative. This can be neutralized by placing an amplifier with negative gain at the output of the weighted adder.




The output of the integrator is

$$\int^{t} x_n(dt')dt'$$

If the message signal is a sinusoid, the integrator output is a sinusoid having the same frequency; but the (input and output) amplitudes are different and the phases are different.

The Armstrong phase modulator looks similar to a modulator for AM. For both modulators, a double-sideband term and a residual carrier are added together. Furthermore, the spectrum of the output from the Armstrong modulator looks similar to the spectrum of AM. For a sinusoidal message signal, there are three spectral lines: one from the residual carrier and two from the double-sideband term. However, an Armstrong modulator does not produce an AM carrier. In the Armstrong modulator the phase shifter (which delays the residual carrier by $\pi/2$ radians) makes the difference.

In the remainder of this section, the assumption is made that $x_n(t)$ is $\cos(2\pi f_m t)$. Noting that $\cos(2\pi f_c t)$ with a delay of $\pi/2$ radians is $\sin(2\pi f_c t)$, the output of the Armstrong modulator is of the form

$$\begin{aligned} &a\sin(2\pi f_c t) + b\sin(2\pi f_m t)\cos(2\pi f_c t) = \\ &\quad \sqrt{a^2 + b^2\sin^2(2\pi f_m t)} \cdot \sin[2\pi f_c t + \tan^{-1}(b\sin(2\pi f_m t)/a)] \end{aligned} \tag{1}$$

This signal shows both amplitude and angle modulation.

If $|b|$ is small compared with $|a|$, then Eq. (1) can be approximated as

$$\begin{aligned} &\sqrt{a^2 + b^2\sin^2(2\pi f_m t)} \cdot \sin[2\pi f_c t + \tan^{-1}(b\sin(2\pi f_m t)/a)] \cong \\ &\qquad |a|\sin[2\pi f_c t + \beta\sin(2\pi f_m t)] \end{aligned} \tag{2}$$

where

$$\beta = \tan^{-1}\left(\frac{b}{a}\right) \tag{3}$$

$\beta$ is the modulation index discussed previously. Eq. (2) has the form of FM with a sinusoidal message signal.

One method of setting the modulation index $\beta$ is to individually set the parameters $a$ and $b$ such that they are related to the desired value of $\beta$ through Eq. (3). In the laboratory it is generally more convenient to measure the rms value of signals, rather than the peak values ($|a|$ and $|b|$). The relationship between rms and peak values depends on the type of signal. The table below shows these relationships for the signals of interest here.

| Signal | Example | rms |
|---|---|---|
| sinusoid | $a\sin(2\pi f_c t)$ | $\lvert a\rvert/\sqrt{2}$ |
| product of sinusoids | $b\sin(2\pi f_m t)\cos(2\pi f_c t)$ | $\lvert b\rvert/2$ |

The following rms voltages can be measured:

| | |
|---|---|
| $V_{\mathrm{DSB}}$ | rms voltage of double-sideband (DSB) component at modulator output |
| $V_{\mathrm{C}}$ | rms voltage of residual carrier (C) component at modulator output |

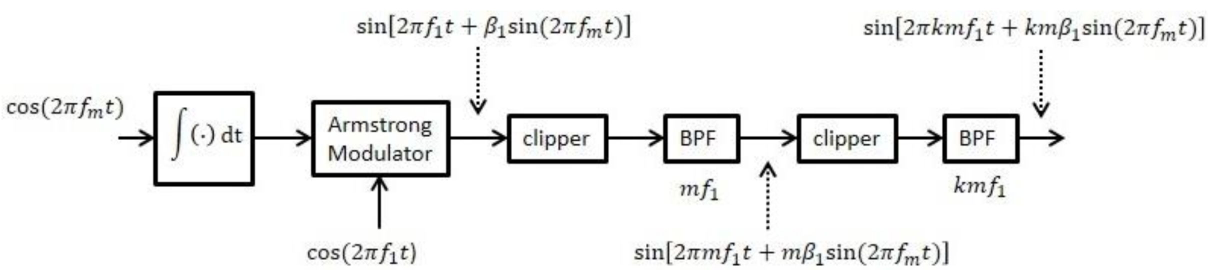
$V_{\mathrm{DSB}}$ can be measured by temporarily disconnecting the residual carrier from the modulator. Similarly, $V_{\mathrm{C}}$ can be measured by temporarily disconnecting the DSB component from the modulator (so that only the residual carrier is present). The modulation index can then be calculated as

$$\beta = \tan^{-1}\left(\frac{\sqrt{2}\,V_{\mathrm{DSB}}}{V_{\mathrm{C}}}\right) \tag{4}$$

The generation of FM by the indirect method, employing an integrator and phase modulator, has an important practical advantage over the direct method. A VCO typically has poor frequency stability, so a carrier generated from a VCO (direct method) exhibits frequency drift. With the indirect method a very stable oscillator (usually a crystal oscillator) can be used as the source of the carrier.

0.2 Frequency Multiplication

An integrator followed by an Armstrong phase modulator can produce a good approximation of FM when the desired modulation index $\beta$ is small. This is called *narrowband FM*, owing to the fact that the modulation index and therefore the bandwidth is small. However, the well-known performance advantage of FM over AM occurs only when $\beta$ is large. However, there is a way to use an Armstrong modulator and yet still achieve a large $\beta$. This is illustrated below. (For the sake of having a definite example, the illustration below assumes the message signal is a sinusoid.)




In words, the FM is generated with an integrator followed by an Armstrong phase modulator. The carrier frequency $f_1$ and the modulation index $\beta_1$ at the output of the Armstrong modulator are both smaller than desired. This narrowband FM signal is placed at the input of a (memoryless) nonlinear device (such as a clipper), which generates harmonics. Then the *m*-th harmonic is selected by a bandpass filter, having a passband centered at $mf_1$. The output of this bandpass filter is a carrier of frequency $mf_1$, having FM with modulation index $m\beta_1$. The combination of the nonlinear device and the bandpass filter is called a *frequency multiplier*. The truth be told, *angle multiplier* would be a better name, because it is the angle (the argument of the carrier sinusoid) that is multiplied. A second frequency multiplier can be used to further increase both the modulation index and the carrier frequency. If the second frequency multiplier employs a factor $k$ (selects the $k$-th harmonic), then the output of this second frequency multiplier is a carrier of frequency $kmf_1$, having FM with modulation index $km\beta_1$. The following equations summarize the effect of the frequency multiplications:

$$f_c = kmf_1 \tag{5}$$

$$\beta = km\beta_1 \tag{6}$$

In this way, quite large modulation indices can be achieved. Because this is the indirect method for generating FM, which permits use of a crystal oscillator for the frequency $f_1$, the carrier can have excellent frequency stability.

## 1 Armstrong Modulator and Frequency Multiplication

You will build an Armstrong (phase) modulator. With an integrator placed before the Armstrong modulator, the combination becomes an indirect method for generating FM. You will begin by experimenting with the integrator. In the first Armstrong modulator that you build, the carrier frequency will be 100 kHz.

The combination of integrator and Armstrong modulator produces a good approximation to FM only for small modulation indices. You will demonstrate that this limitation can be overcome by following the Armstrong modulator with frequency multipliers. These devices multiply the modulation index as well as the carrier frequency. In this experiment, you will use a cascade of two frequency multipliers, each with multiplication factor 3. Therefore, the modulation index at the output of the entire chain will be 9 times the modulation index produced at the immediate output of the Armstrong modulator. The carrier frequency also gets multiplied by 9, so the carrier used within the Armstrong modulator must be (100/9) kHz if the carrier at the output of the entire chain is to be 100 kHz.

### 1.1 Integrator

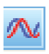
You will use the $s^{-1}$ function in the Laplace Transform module to perform the integration required before the Armstrong (phase) modulator in order to achieve a frequency modulator. A linear time-invariant system having a transfer function $s^{-1}$ is an indefinite integrator. However, the so-called $s^{-1}$ function in the Laplace Transform module actually produces

$$C - \int^{t} x(t')\, dt'$$

at its output, where $C$ is a constant and $x(t)$ is the analog input. The output of the Laplace module will go to a multiplier that will be set for AC coupling, and that will eliminate the unwanted DC component $C$. The negative sign in front of the indefinite integral is not desirable; but, on the other hand, it won't be a real problem. The message signal $x(t)$ will be a sinusoid here, so the negative sign just shifts the phase of the message sinusoid by $180^\circ$.

The indefinite integral of a sinusoid is a sinusoid of the same frequency but with a different amplitude (which is inversely proportional to the frequency) and a different phase. The phase change is $-\pi/2$ radians for an indefinite integral. For an indefinite integral with a negative sign in front, the phase change is $\pi - \pi/2 = +\pi/2$ radians. That is to say, the output sinusoid will lead the input by $90^\circ$.

Place a sinusoid from the Audio Oscillator on the input of the (Laplace Transform module's) $s^{-1}$ function and also on Channel A of the oscilloscope. Place the output on Channel B. Since this integrator has a negative sign, the output should lead the input by $90^\circ$. Try a few different frequencies for the sinusoid produced by the Audio Oscillator. You should observe the expected phase relationship between the Channel A and B sinusoids.


Channel A: input of $s^{-1}$ function (for one example frequency)
Channel B: output of $s^{-1}$ function

Switch the oscilloscope to X-Y View

**`Views > X-Axis > A`**

You will observe an ellipse. If the phase difference between the two sinusoids is truly $90^\circ$, the one axis of the ellipse will be horizontal and the other vertical.


Channel A: input of $s^{-1}$ function (for one example frequency)
Channel B: output of $s^{-1}$ function

As you change the frequency of the input sinusoid, you should observe that the relative lengths of the axes of the ellipse change, since the amplitude of the integrator output is inversely proportional to the frequency. But the axes should remain horizontal and vertical.

### 1.2 Armstrong Modulator

Construct an Armstrong (phase) modulator. Place the output of the (Laplace Transform module's) $s^{-1}$ function at the input of the Armstrong modulator. Set the Multiplier within the Armstrong modulator for AC coupling, in order to block the (unwanted) DC component on the arriving signal. The output of the Armstrong modulator (with preceding integrator) will approximate FM when the modulation index is small.

For now use a 100-kHz sinusoid (Master Signals) as the unmodulated carrier for the Armstrong modulator. (This will change later.) Set the frequency-range switch on the Phase Shifter PCB to "HI". Adjust the delay of the Phase Shifter so that it changes the phase of a 100-kHz sinusoid by $-\pi/2$ radians.

Include a (negative-gain) Buffer Amplifier after the (weighted) Adder, in order to cancel the (negative) weighting factors of the Adder.

The 2-kHz analog sinusoid (Master Signals) will serve as the message signal. Therefore, it should be placed at the input to the integrator.

Adjust the modulation index $\beta$ to 0.5. You can accomplish this with the help of Eq. (4), recognizing that $V_{\mathrm{C}}$ is the rms voltage at the modulator output with only the residual-carrier component present and $V_{\mathrm{DSB}}$ is the rms voltage with only the DSB component present. You might approach this by rewriting Eq. (4) like so:

$$\frac{V_{\mathrm{DSB}}}{V_{\mathrm{C}}} = \frac{\tan(\beta)}{\sqrt{2}} \tag{7}$$

This requires the calculation of $\tan(\beta)$. $\beta$ must be treated as an angle in radians.

Ask your instructor to verify your procedure for setting $\beta$.

Connect the Armstrong modulator output to Channel A. You should observe that both frequency and amplitude modulation are present. With this small modulation index, however, the amplitude modulation should be relatively small.

Channel A: Armstrong modulator output

Now observe the spectrum of the Armstrong modulator output. Measure $P_1/P_0$. Compare this measurement with what might be expected, based on the $\beta = 0.5$ row of Table 1.

Channel A: Armstrong modulator output

Table 1: $P_1/P_0$ as a Function of $\beta$ for a Sinusoidal Message

| $\beta$ | $P_1/P_0$ (dBc) |
|---|---|
| 0.5 | $-11.8$ |
| 1.0 | $-4.8$ |
| 1.5 | 0.7 |
| 2.0 | 8.2 |
| 2.5 | 20.2 |
| 3.0 | 2.3 |

## 1.3 Frequency Multipliers

You will build two frequency multipliers. For this purpose you will need the 33.3-kHz bandpass filter on the FM Utilities module and the 100-kHz bandpass filter on the 100-kHz Channel Filters module, with the module switch in Position 3.

Obtain a quick view of the magnitude of the transfer function for each of these filters. You can do this by connecting the Noise Generator module to the input of a filter and then observing the spectrum of the filter output.

Channel A: 33.3-kHz BPF output, showing $|H(f)|$

Channel A: 100-kHz BPF output, showing $|H(f)|$

Use the FM Utilities module to create a stable 11.1-kHz sinusoid. You can do this by providing a 100-kHz sinusoid (Master Signals) to the input port that expects a 100-kHz sinusoid. The adjacent output port then supplies a sinusoid with a frequency of (100/9) kHz, approximately 11.1 kHz.

In the Armstrong modulator, replace the 100-kHz unmodulated carrier with the 11.1-kHz sinusoid. It is essential that you readjust the delay of the Phase Shifter so that a 11.1-kHz sinusoid experiences a phase change of $-\pi/2$ radians. (Recall that the Phase Shifter introduces a phase delay that depends on the frequency.)

The integrator should remain in place (providing the input to the Armstrong phase modulator). The input to the integrator should be the 2-kHz analog sinusoid (Master Signals).

The output of this Armstrong modulator is now an 11.1-kHz carrier with FM. In the experimentation that follows, the modulation index at the output of the Armstrong modulator will be set to several different values; whatever the numeric value, we will denote this modulation index by $\beta_1$.

Connect the output of the Armstrong modulator to a sequence of two frequency multipliers, each multiplying the frequency and modulation index by 3. The first frequency multiplier will consist of a clipper followed by the 33.3-kHz BPF. The output of the first frequency multiplier will be a carrier of frequency $3 \times 11.1$ kHz $= 33.3$ kHz with a modulation index of $3\beta_1$. The second frequency multiplier will consist of a clipper followed by the 100-kHz BPF. The output of the second frequency multiplier will therefore be a carrier of frequency 100 kHz with a modulation index of $9\beta_1$. The two clippers are available on the FM Utilities module.

The modulation index at the output of the Armstrong modulator is $\beta_1$, and the modulation index at the output of the second frequency multiplier is $\beta_9$. Therefore,

$$\beta_9 = 9\beta_1 \tag{8}$$

You will be setting $\beta_9$ to 6 different values and, for each numeric value of $\beta_9$, making measurements of the spectrum at the second frequency multiplier output (which represents the output of the complete frequency modulator). The measurements will be recorded in an Excel worksheet.

frequency multipliers

Use the following two-step procedure to set each value of $\beta_9$.

1. Calculate $\beta_1$ from $\beta_9$ with the aid of Eq. (8).
2. Set $\beta_1$ in the Armstrong modulator with the help of Eq. (4) or, equivalently, Eq. (7).

For each numeric value of $\beta_9$, you will make measurements. Don't change the value of $\beta_9$ until you've completed all measurements, described below, for each value of $\beta_9$.

Observe the spectrum at the output of the Armstrong modulator.

Channel A: Armstrong modulator output
Capture this spectrum for each value of $\beta_9$. (That's a total of 6 spectra.)

Observe the spectrum at the output of the first frequency multiplier (where the carrier frequency is 33.3 kHz).

Channel A: 33.3-kHz carrier
Capture this spectrum for each value of $\beta_9$. (That's a total of 6 spectra.)

Observe the spectrum at the output of the second frequency multiplier (where the carrier frequency is 100 kHz).

Channel A: 100-kHz carrier
Capture this spectrum for each value of $\beta_9$. (That's a total of 6 spectra.)

It should be clear from the spectra that the modulation index increases with each frequency multiplication.

At the output of the second frequency multiplier, measure the line height of the residual carrier (in dBu) and of the upper fundamental (in dBu). Compute $P_1/P_0$ (in dBc) based on your measurements. You should compare your measurement results with the predictions (based on theory) of Table 1.

Plot the measured value of $P_1/P_0$ (in dBc) as a function of $\beta_9$, using discrete points. Also plot (on the same set of axes) the theoretical value of $P_1/P_0$ (in dBc) as a function of $\beta_9$, using discrete points. The theoretical value is given in Table 1. (For present purposes, the modulation index of Table 1 is interpreted as $\beta_9$.)